inkMaths

ANSWER BOOK

The Poisson distribution

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Events must occur singly, at random, independently, and at a constant average rate over the window in question. The only parameter is that mean rate λ, which is also the variance. [2]

A2 $P(X = 3) = e^{-4} \times 4^3/3! = 0.1954$. $P(X \le 2) = e^{-4}(1 + 4 + 8) = 0.2381$. The cumulative value comes fastest from a calculator's own Poisson function. [3]

SECTION B · Standard

B1 Scale λ with the window: over two minutes the count is $Po(2.5 \times 2) = Po(5)$. So $P(X = 0) = e^{-5} = 0.0067$, and the variance is 5, equal to the mean. Rescaling λ to the interval actually asked about is the step that decides the whole answer. [4]

B2 $P(X \ge 8) = 1 - P(X \le 7) = 1 - 0.7440 = 0.2560$. Converting an 'at least' to one minus an 'at most', with the boundary dropped by one, is the standard manoeuvre. [3]

B3 n is large and p small, so $B(200, 0.01) \approx Po(2)$ with $\lambda = np = 2$. Then $P(X \le 1) = e^{-2}(1 + 2) = 0.4060$. The exact binomial gives 0.4046, so the approximation is out by 0.0014: close enough for any practical purpose. [4]

SECTION C · Stretch

C1 The additive property pools them to $Po(3.5)$ per hour, and scaling the window to three hours gives $Po(10.5)$. Both the mean and the variance are 10.5, because a Poisson distribution has only one parameter and it serves as both. That equality is also the test of whether the model fits real data. [4]