InkMaths

ANSWER BOOK

The product, quotient and chain rules

Differentiation · Year 12-13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Outside first, leaving the inside alone: $4(2x + 1)^3$, then multiply by the inside's derivative, 2: $dy/dx = 8(2x + 1)^3$. Rates through a chain multiply. [2]

A2 The outside is e to something, its own derivative; the inside x^2 contributes 2x. So $dy/dx = 2x\,e^{x^2}$. The exponential never changes shape; the chain factor does all the varying. [2]

SECTION B · Standard

B1 With $u = x^2 - 3$: $dy/du = 6u^5$ and $du/dx = 2x$, so $dy/dx = 12x(x^2 - 3)^5$. On show-your-working questions the u earns its ink; with practice it goes silent. [3]

B2 Product rule with $u = x^3$, $v = e^{2x}$: $dy/dx = 3x^2e^{2x} + 2x^3e^{2x} = x^2e^{2x}(3 + 2x)$, with the factorised form the tidier finish. Each factor takes a turn being differentiated while the other watches. [3]

B3 Product rule: $dy/dx = 1 \times \sin x + x \times \cos x = \sin x + x \cos x$. Differentiating each factor and multiplying the results would give cos x alone, which is wrong: the rule shares the differentiation, one factor at a time. [3]

B4 Quotient rule with $u = 2x + 1$, $v = x^2 + 1$: $dy/dx = [2(x^2 + 1) - (2x + 1)(2x)]/(x^2 + 1)^2 = (-2x^2 - 2x + 2)/(x^2 + 1)^2$. At $x = 0$ the gradient is 2. The minus sign in the numerator makes the order u′v - uv′ unforgiving: swap it and every answer flips sign. [4]

SECTION C · Stretch

C1 Read it as $(\sin x)^3$: outside gives $3(\sin x)^2$, inside gives cos x, so $dy/dx = 3\sin^2 x \cos x$. The notation $\sin^3 x$ hides the chain; rewriting with the bracket makes the two layers visible. [2]