



ANSWER BOOK

The quality of tests

Further Statistics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** A Type I error rejects H_0 when H_0 is true: a false alarm. A Type II error fails to reject H_0 when a particular alternative is true: a missed change. The first needs no alternative specified; the second cannot be computed without one. [2]
- A2** Size = $P(X \leq 1 \mid p = 0.5) = (1 + 10)/2^{10} = 11/1024 = 0.0107$. That is well below a nominal 5%, [3] because the discrete jump from $X = 1$ to $X = 2$ overshoots the level.

SECTION B · Standard

- B1** $P(X \leq 1 \mid p = 0.2) = 0.8^{10} + 10(0.2)(0.8^9) = 0.1074 + 0.2684 = 0.3758$, so the test rejects with probability 0.3758: that is the power. The Type II probability is $1 - 0.3758 = 0.6242$, so the test misses a drop to 0.2 nearly two times in three. [4]
- B2** $P(X \leq 1 \mid p = 0.1) = 0.9^{10} + 10(0.1)(0.9^9) = 0.3487 + 0.3874 = 0.7361$. The power has almost doubled: the further the truth lies from H_0 , the more easily the test detects it. [3]
- B3** Size = $P(X \geq 10 \mid \lambda = 5) = 1 - P(X \leq 9) = 0.0318$. Power at $\lambda = 8$ is $P(X \geq 10 \mid \lambda = 8) = 1 - P(X \leq 9) = 0.2834$. Even a sizeable rise in the rate is caught only about a quarter of the time by a single observation. [4]

SECTION C · Stretch

- C1** A wider region rejects H_0 more readily, so it catches more true departures and the power rises at every alternative. It also rejects more often when H_0 is true, so the size rises with it: more false alarms. With the sample size fixed the two move together, and the right balance depends on which error costs more. Only a larger sample improves both at once. [4]