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ANSWER BOOK

The Simplex algorithm

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Stop when there are no negative entries left in the objective row. A negative entry means increasing that variable would still improve the objective, so while one remains the solution can be bettered. Once none remain, no variable can be increased without reducing the objective, so the current corner is optimal. [2]

A2 Add slack variables: $x + y + s_1 = 10$ and $2x + 3y + s_2 = 24$, with the objective as $P - 4x - 5y = 0$. The tableau rows are P: 1, -4, -5, 0, 0 | 0; s_1: 0, 1, 1, 1, 0 | 10; s_2: 0, 2, 3, 0, 1 | 24. [3]

SECTION B · Standard

B1 The most negative entry in the objective row is -5, so the pivot column is y. Ratio test: $10 \div 1 = 10$ and $24 \div 3 = 8$, so the pivot row is the s_2 row. Divide that row by 3, then subtract it from the s_1 row and add five times it to the objective row. That gives P = 40 with y = 8, and the objective row still has a negative entry in x. [4]

B2 No entry in the objective row is negative, so no variable can be increased to improve P; the tableau is optimal. The basic variables are x = 6 and y = 4, the non-basic slacks are $s_1 = s_2 = 0$, and P = 44. Both slacks being zero means both constraints are tight, matching the graphical answer. [4]

B3 Ratios: $12 \div 2$ = 6 and $20 \div 4$ = 5. The -1 is skipped entirely: increasing the entering variable would make that row's value grow rather than shrink, so it places no limit. The smallest ratio is 5, so the third row is the pivot row. [3]

SECTION C · Stretch

C1 With only ≤ constraints the origin is feasible and the slack variables form a valid starting solution. A ≥ constraint excludes the origin, so setting all the real variables to zero gives a negative surplus and no starting corner. The two-stage method first minimises the sum of the artificial variables, driving them to zero and so reaching a feasible corner, then runs ordinary Simplex from there. The big-M method instead subtracts a large multiple M of each artificial variable from the objective, making them so expensive that the algorithm eliminates them on its own. [4]

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