



QUESTION WORKBOOK

The structure of proof: deduction and exhaustion

Proof · Year 12–13

8 questions · 22 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

- A1** Write down general algebraic forms for: an even number, an odd number, and a pair of consecutive odd numbers. In each case say what the letter stands for.

[2]

- A2** A student checks that a claim holds for $n = 1$ to $n = 50$ and writes “proved”. Explain why the work is not a proof, and name the kind of argument that would be.

[2]

SECTION B · Standard

- B1** Prove that $n^2 + 4n + 7 > 0$ for all values of n .

[3]

- B2** Prove that the sum of any two consecutive odd numbers is divisible by 4.

[3]

- B3** Prove that the difference between the squares of any two consecutive odd numbers is divisible by 8. [3]

- B4** Prove that every square number is either a multiple of 4 or one more than a multiple of 4. [3]

SECTION C · Stretch

- C1** Given that p is a prime number with $5 < p < 20$, prove by exhaustion that $p^2 - 1$ is divisible by 24. [4]

- C2** Explain why proof by exhaustion could not be used to prove that $n^2 + 4n + 7 > 0$ for all integers n , even in principle. [2]