inkMaths

ANSWER BOOK

The t-formulae

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\sin\theta = 2t/(1 + t^2)$, $\cos\theta = (1 - t^2)/(1 + t^2)$ and $\tan\theta = 2t/(1 - t^2)$. All three share the same right triangle, with legs $2t$ and $1 - t^2$ and hypotenuse $1 + t^2$. [2]

A2 $1 + t^2 = 5$, so $\sin\theta = 4/5$ and $\cos\theta = (1 - 4)/5 = -3/5$. A negative cosine with a positive sine puts θ in the second quadrant, which a sketch confirms since $t > 1$ means $\theta/2$ exceeds 45°. [3]

SECTION B · Standard

B1 Substituting and multiplying by $1 + t^2$: $3(1 - t^2) - 8t = 5(1 + t^2)$, so $8t^2 + 8t + 2 = 0$, that is $(2t + 1)^2 = 0$. The repeated root $t = -1/2$ gives $\theta = 2\arctan(-1/2) = -0.927$, which is 5.36 radians in the required interval. A repeated root was inevitable: $\sqrt{3^2 + 4^2} = 5$ is the expression's maximum magnitude, so the level is reached tangentially. [4]

B2 $\tan(\pi/2)$ is undefined, so $\theta = \pi$ corresponds to no finite value of t and the substitution simply cannot reach it. Whenever the interval contains π, substitute $\theta = \pi$ into the original equation by hand and check separately whether it is a solution. [3]

B3 With $t = \tan 22.5°$, the formula gives $\tan 45° = 2t/(1 - t^2) = 1$, so $2t = 1 - t^2$, that is $t^2 + 2t - 1 = 0$. Solving: $t = (-2 \pm \sqrt{8})/2 = -1 \pm \sqrt{2}$. Since 22.5° is acute, t is positive, so $t = \sqrt{2} - 1 \approx 0.414$. [4]

SECTION C · Stretch

C1 Write the left side over $\sin\theta$ and $\cos\theta$: $(1 + 1/\sin\theta)/(\cos\theta/\sin\theta) = (\sin\theta + 1)/\cos\theta$. Now substitute the t-formulae: the numerator becomes $(2t + 1 + t^2)/(1 + t^2) = (1 + t)^2/(1 + t^2)$ and the denominator $(1 - t^2)/(1 + t^2) = (1 - t)(1 + t)/(1 + t^2)$. Dividing, the $(1 + t^2)$ and one factor of $(1 + t)$ cancel, leaving $(1 + t)/(1 - t)$ as required. [4]