inkMaths

ANSWER BOOK

The travelling salesman problem

Decision Mathematics 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 The practical problem allows a node to be visited more than once; the classical problem visits each exactly once and needs a complete network obeying the triangle inequality. Replacing every pair's distance by the shortest path between them produces such a network, turning a practical problem into a classical one. [2]

A2 From A the nearest is B (20). From B the nearest unvisited is E (28), then D (22), then C (12). Returning C to A costs 35. Tour A B E D C A, length 117. [3]

SECTION B · Standard

B1 Removing A leaves B, C, D, E. Their minimum spanning tree takes CD 12, DE 22 and BE 28, total 62. The two shortest edges from A are AB 20 and AE 25, total 45. Lower bound = 107. No tour can be shorter, because every tour contains a spanning structure of exactly that form. [4]

B2 20 + 30 + 12 + 22 + 25 = 109, which is a better upper bound than the 117 nearest neighbour gave. So the optimal tour T satisfies 107 ≤ T ≤ 109. Nearest neighbour was eight worse than this tour: it is quick and always gives a valid tour, but it is not reliable. [4]

B3 Each deletion gives a valid lower bound, but the bounds differ because the trees and the two shortest edges differ. Since every one is a genuine lower bound, the largest of them is the strongest statement available, so try several deletions and quote the largest result. [3]

SECTION C · Stretch

C1 Lower bound = 48 + 9 + 11 = 68, and the tour gives an upper bound of 71, so the optimal tour T satisfies 68 ≤ T ≤ 71. The gap is only three, so the tour found is at worst three longer than the best possible. If a tour of exactly 68 were found the bounds would meet and it would be provably optimal. [4]