inkMaths

ANSWER BOOK

The vector product and the scalar triple product

Further Pure 1 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $|\mathbf{a} \times \mathbf{b}|$ is the area of the parallelogram spanned by a and b, equal to $|\mathbf{a}||\mathbf{b}| \sin \theta$. The scalar triple product $\mathbf{a}\cdot\mathbf{b} \times \mathbf{c}$ is the volume of the parallelepiped on the three vectors. [2]

A2 $\mathbf{a} \times \mathbf{b} = ((-1)(-2) - (3)(4), (3)(1) - (2)(-2), (2)(4) - (-1)(1)) = (-10, 7, 9)$. Dot with a: $-20 - 7 + 27 = 0$, so the product is perpendicular to a, as it must be. Dotting with b gives 0 too, and both checks take seconds. [3]

SECTION B · Standard

B1 $AB = (2, -1, 3)$ and $AC = (1, 4, -2)$. Their cross product is $(-10, 7, 9)$, with magnitude $\sqrt{(100 + 49 + 81)} = \sqrt{230} \approx 15.17$. The triangle is half the parallelogram, so its area is $\sqrt{230}/2 \approx 7.58$. [4]

B2 $\mathbf{b} \times \mathbf{c} = ((3)(4) - (1)(2), (1)(0) - (2)(4), (2)(2) - (3)(0)) = (10, -8, 4)$. Then $\mathbf{a}\cdot(\mathbf{b} \times \mathbf{c}) = 10 - 8 + 4 = 6$, the parallelepiped's volume. The tetrahedron takes a sixth of it, so its volume is 1. [4]

B3 The length is $\sqrt{(1 + 4 + 4)} = 3$, so the direction cosines are $1/3$, $2/3$ and $2/3$. Squaring and adding: $(1 + 4 + 4)/9 = 1$, as the cosines of the angles a direction makes with the three axes must always satisfy. [3]

SECTION C · Stretch

C1 A zero triple product means the parallelepiped has no volume, so the three vectors are coplanar: one lies in the plane of the other two. A zero vector product means the parallelogram has no area, so those two vectors are parallel. Both are degeneracy tests, one dimension apart. [4]