



ANSWER BOOK

Transformations of the complex plane

Further Pure 2 · Year FM

6 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The transformation is a translation by the vector $2 - i$, so $z = w - 2 + i$. Substituting gives $|w - 2 + i| = 3$: a circle of the same radius 3, now centred at $2 - i$. Translations move a locus without changing its shape or size.
- A2** Since $2i$ has modulus 2 and argument $\pi/2$, the map is an enlargement scale factor 2 [3] about the origin combined with a rotation of 90° anticlockwise. Substituting $z = w/(2i)$ gives $|w/(2i) - 1| = 1$, and multiplying through by $|2i| = 2$ gives $|w - 2i| = 2$. The centre 1 has moved to $2i$ and the radius has doubled.

SECTION B · Standard

- B1** Write $w = u + iv$, so $z = 1/w = (u - iv)/(u^2 + v^2)$ and $\operatorname{Re}(z) = u/(u^2 + v^2)$. Setting that equal to $1/2$ gives $u^2 + v^2 = 2u$, that is $(u - 1)^2 + v^2 = 1$: a circle of centre 1 and radius 1. It passes through the origin, which is the image of the point at infinity along the line.
- B2** Taking moduli, $|w| = 1/|z| = 1/2$, so the image is the circle $|w| = 1/2$. [4] Since $\arg w = -\arg z$, a point moving anticlockwise round the original circle traces the image clockwise: inversion reflects in the real axis as well as reciprocating the modulus.
- B3** Put $z = 1 + it$, so $w = 1 - t^2 + 2it$. Then $u = 1 - t^2$ and $v = 2t$, so $t = v/2$ and $u = 1 - v^2/4$. [3] Rearranged, $v^2 = 4(1 - u)$: a parabola with vertex at $u = 1$ opening in the negative u direction. Squaring does not keep straight lines straight.

SECTION C · Stretch

- C1** Put $z = \cos \theta + i \sin \theta$. Then $w = (\cos \theta + 1 + i \sin \theta)/(\cos \theta - 1 + i \sin \theta)$; multiplying top and bottom by the conjugate of the denominator makes the denominator $(\cos \theta - 1)^2 + \sin^2 \theta = 2 - 2\cos \theta$, and the real part of the numerator is $(\cos^2 \theta - 1) + \sin^2 \theta = 0$. So w is purely imaginary for every θ . The point $z = 1$ is excluded, since the denominator vanishes there, and no finite w corresponds to it.