



ANSWER BOOK

Trigonometric graphs and equations

Trigonometry · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $\sin 30^\circ = \frac{1}{2}$, $\cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 60^\circ = \sqrt{3}$. All of them come from two triangles: the halved equilateral and the halved square. [2]
- A2** The principal value is 60° , and cosine's symmetry supplies the partner $360^\circ - 60^\circ = 300^\circ$. So $x = 60^\circ$ or 300° : one root each side of the 360° line of symmetry. [2]

SECTION B · Standard

- B1** The calculator's principal value is -30° , outside the interval; the graph puts the actual roots where sine is negative, in the third and fourth quadrants: $x = 180^\circ + 30^\circ = 210^\circ$ and $360^\circ - 30^\circ = 330^\circ$. [3]
- B2** Replace $\sin^2 x$ with $1 - \cos^2 x$: $2 - 2 \cos^2 x + \cos x - 1 = 0$, which tidies to $2 \cos^2 x - \cos x - 1 = 0$ and factorises as $(2 \cos x + 1)(\cos x - 1) = 0$. $\cos x = 1$ gives $x = 0^\circ$; $\cos x = -\frac{1}{2}$ gives $x = 120^\circ$ and 240° . Three solutions: $0^\circ, 120^\circ, 240^\circ$. [3]
- B3** Gather and factorise: $\sin x (\cos x - 2) = 0$. The bracket can never be zero, since cosine never reaches 2, so the only route is $\sin x = 0$: $x = 0^\circ$ or 180° . Dividing both sides by $\sin x$ at the start would have destroyed every solution the equation has. [3]
- B4** Dividing by $\cos x$ is safe here, since $\cos x = 0$ makes the left side ± 3 , not 0: $\tan x = \frac{2}{3}$, so $x = 33.7^\circ$ and, one half-turn along, 213.7° . Tangent repeats every 180° , so a tan equation always gives two roots in a 360° window. [3]

SECTION C · Stretch

- C1** Let $\theta = x - 40^\circ$, so θ runs over $-40^\circ \leq \theta < 320^\circ$ while x covers its interval. $\cos \theta = \frac{1}{2}$ at $\theta = 60^\circ$ and 300° , and also at -60° , which the widened interval excludes ($-60^\circ < -40^\circ$). Translating back: $x = 100^\circ$ or 340° . Widening the interval before hunting roots is the whole discipline; solving first and shifting after loses or invents solutions. [3]