inkMaths

ANSWER BOOK

Trigonometric modelling

Trigonometry · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Amplitude 3; period $2\pi/2 = \pi$ seconds. The 2 inside the bracket speeds the wave up and lives in the period; the 3 outside scales the swing and lives in the amplitude. [2]

A2 The cosine swings between ±1, so h runs from 5 - 2 = 3 up to 5 + 2 = 7. The 5 is the centre line, the 2 the swing about it. [2]

SECTION B · Standard

B1 The axle sits on the centre line, 10 m up, and the radius is the amplitude, 8 m. The seat tops out at 10 + 8 = 18 m and bottoms at 2 m. The period is $2\pi \div (\pi/20) = 40$ s. The minus sign starts the seat at the bottom of the wheel at t = 0, where a ride starts. [4]

B2 $10 - 8\cos(\pi t/20) = 14$ gives $\cos(\pi t/20) = -\frac{1}{2}$, so the first positive solution is $\pi t/20 = 2\pi/3$, and $t = 40/3 = 13.3$ s to 3 significant figures. Later crossings exist, but the question wants the first: principal value, then translate. [3]

B3 Amplitude 0.9 m; period $2\pi/0.6 = 10.5$ s. Mean sea level is y = 0, and after t = 0 the sine next vanishes at $0.6t = \pi$, so t = 5.24 s: half a period after launch, as the graph shows. [3]

SECTION C · Stretch

C1 $R = \sqrt{(4 + 2.25)} = 2.5$ and $\tan\alpha = 1.5/2$ gives $\alpha = 0.6435$, so $y = 2.5\sin(0.5t + 0.6435)$. High tide is 2.5 m, where the sine reaches 1: $0.5t + 0.6435 = \pi/2$, so t = 1.85 hours. Two terms, one tide: the harmonic form is the tool that makes the data readable. [4]

C2 Amplitude and period are separate dials. Doubling the amplitude doubles how far the oscillation swings and leaves its timing alone; oscillating faster needs a bigger coefficient inside the bracket, which shortens the period. The claim muddles the outside dial with the inside one. [2]