inkMaths

ANSWER BOOK

Vectors in three dimensions

Vectors · Year 12–13

7 questions · 20 marks

NAVY EDITION

inkmaths.co.uk

SECTION A · Warm-up

A1 $|6i - 2j + 3k| = \sqrt{(36 + 4 + 9)} = \sqrt{49} = 7$. Three squares under one root: the two-dimensional formula with one more term. [2]

A2 The unit vector is $(6/7)i - (2/7)j + (3/7)k$. For magnitude 21, scale by $21/7 = 3$: $18i - 6j + 9k$, or its negative $-18i + 6j - 9k$, since parallel allows either sense. [2]

SECTION B · Standard

B1 $AB = b - a = 2i - 4j + 6k$. The distance is $|AB| = \sqrt{(4 + 16 + 36)} = \sqrt{56} = 2\sqrt{14}$, left exact as asked. The midpoint averages the coordinates: $(2, 0, 2)$. [3]

B2 $a^2 + 16 + 64 = 144$, so $a^2 = 64$ and $a = \pm 8$. Both signs survive, because squaring hides the sign: the answer is two vectors, mirror images in the jk-plane. [2]

B3 $PQ = 2i + 4j + 2k$ and $PR = 5i + 10j + 5k$, so $PR = 2.5PQ$: a scalar multiple. The two vectors are parallel and share the start point P, so the three points lie on one line. In three dimensions there is no single cross-multiplication check; the multiple must work in every component. [3]

SECTION C · Stretch

C1 $AB = i + 2j + 2k$ with $|AB| = \sqrt{9} = 3$, and $AC = 2i - 2j + k$ with $|AC| = \sqrt{9} = 3$: isosceles. $BC = C - B = i - 4j - k$, so $|BC|^2 = 1 + 16 + 1 = 18$. Since $|AB|^2 + |AC|^2 = 9 + 9 = 18 = |BC|^2$, the converse of Pythagoras puts the right angle at A, between the two equal sides. [4]

C2 First run Pythagoras across the floor: the diagonal of the base rectangle has length $\sqrt{(2^2 + 3^2)} = \sqrt{13}$. Then run it again up the wall, on the right-angled triangle made by that diagonal and the vertical component: $\sqrt{(13 + 36)} = \sqrt{49} = 7$. The nested roots collapse into the single formula $\sqrt{(x^2 + y^2 + z^2)}$. [2]