inkMaths

ANSWER BOOK

Vectors in two dimensions

Vectors · Year 12–13

8 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $|a| = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$. The unit vector is a divided by its length: $(5/13)i + (12/13)j$. Its own magnitude is 1 by construction, which is the quick self-check. [2]

A2 $2p + 3q = (4 - 3)i + (-6 + 12)j = i + 6j$. Slot-by-slot arithmetic; the only traps are the signs. [2]

A3 $|7i - 24j| = \sqrt{(49 + 576)} = \sqrt{625} = 25$. The angle satisfies $\tan\theta = 24/7$, so $\theta = 73.7°$ below the positive x-direction, since the j-component is negative. Saying which side of the axis is part of the answer: a magnitude and a bare angle do not pin the vector down. [2]

SECTION B · Standard

B1 $AB = b - a = (4 - (-2))i + (-5 - 3)j = 6i - 8j$: destination minus start. The distance is $|AB| = \sqrt{(36 + 64)} = 10$. The midpoint averages the position vectors: $((-2 + 4)/2, (3 - 5)/2) = (1, -1)$. [3]

B2 Parallel means one is a scalar multiple of the other: $6i + \lambda j = k(4i + 6j)$. The i-components give $k = 6/4 = 3/2$, so $\lambda = 6 \times 3/2 = 9$. Equivalently the components are in proportion, $4/6 = 6/\lambda$, which cross-multiplies to $4\lambda = 36$. [3]

B3 $|5i - 12j| = 13$, and $26 = 2 \times 13$, so scale by ± 2: the vectors are $10i - 24j$ and $-10i + 24j$. Forgetting the negative one loses a mark: parallel includes pointing the opposite way. [3]

SECTION C · Stretch

C1 $a + b = 9i + 12j$, so $|a + b| = \sqrt{(81 + 144)} = 15$, while $|a| + |b| = 5 + 10 = 15$. Equality holds exactly when a and b point the same way (here $b = 2a$): the tip-to-tail path is straight, so the shortcut and the detour are the same walk. [2]

C2 $AB = 3i + 6j$ and $AC = 5i + 10j$, so $AC = (5/3)AB$: a scalar multiple. AB and AC are parallel and both start at A, so the three points are collinear. The shared start point is essential; parallel vectors alone would only give parallel lines. [3]