inkMaths

ANSWER BOOK

Volumes of revolution

Further calculus · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 $V = \pi\int y^2\,dx$ from a to b. Each strip sweeps a disc of radius y and thickness dx, and a disc's volume is π times radius squared times thickness. [2]

A2 $V = \pi\int 9\,dx$ from 0 to 4 = 36π. The solid is a cylinder of radius 3 and length 4, and $\pi r^2 h = \pi \times 9 \times 4 = 36\pi$ agrees. Constant y is the quickest sanity check the formula ever gets. [3]

SECTION B · Standard

B1 $V = \pi\int x^4\,dx$ from 0 to 2 = $\pi[x^5/5] = 32\pi/5$. Square first, integrate second: the integrand is x^4, not x^2, and the power-rule slip of integrating y before squaring is the topic's classic error. [4]

B2 Radius is x with $x^3 = y$, so $x^2 = y^{(2/3)}$ and $V = \pi\int y^{(2/3)}\,dy$ from 0 to 8 = $\pi[3y^{(5/3)}/5] = \pi \times 3 \times 32/5 = 96\pi/5$. About the y-axis everything transposes: y-limits, and x^2 expressed in terms of y. [4]

B3 Outer radius x, inner radius x^2: $V = \pi\int(x^2 - x^4)\,dx$ from 0 to 1 = $\pi(1/3 - 1/5) = 2\pi/15$. Subtract the squares, never square the difference: $\pi\int(x - x^2)^2\,dx$ describes a different and wrong solid. [4]

SECTION C · Stretch

C1 Each cross-section is an annulus: a disc of the outer radius with a disc of the inner radius removed, area $\pi(\text{outer}^2 - \text{inner}^2)$. Squaring the difference would compute a disc whose radius is the gap between the curves, which is a different region rotated about a different axis of symmetry. [3]